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TECHNICAL NOTE

The pressure and velocity fields in the wick structure of a localized heated flat plate heat pipe

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1. INTRODUCTION

Heat pipes have been widely used as heat transmission devices in the last few decades due to their abilities of transporting heat at high rate over considerable distance with small temperature drop [1, 2]. They are found particularly useful as cooling means for modern electronic devices which have been manufactured for high performance and high degree of integration. Inside a heat pipe, working fluid is evaporated at the heat input section or zones, and condensed at the condensation section. A sufficient capillary pressure is needed to balance the gravitational pressure and the pressure losses in both vapor and liquid phases. In general, the calculation of pressure loss in the liquid phase is usually done by using Darcy's equation, and the solution can easily be obtained if both heater and condenser cover the whole evaporation and condensation sections. There are not many solutions available for the cases of localized heating or cooling. Under these situations, the flow patters of liquid in the wick structure are no longer one-dimensional (1D) due to the non-uniform heat flux caused by discrete heaters or condensers, and the pressure distribution are usually difficult to be determined. For a circular heat pipe, Schmalhofer and Faghri [3] considered the case of partially circumferentially-heated condition and presented an approximate method to calculate the effective length. Sun *et al.* [4] applied a similar approach and presented the results for a flat plate heat pipe, in which the heater is not completely covered in the whole evaporation section. Flat plate heat pipes have been proved to be useful for cooling electronic devices [5, 6]. The heat sources applied on the surface of the heat pipes may be discrete. It is worthwhile having some methods to calculate the pressure distribution of the liquid under this condition, which motivated the work presented in this paper.

An analytical method is developed in the present study to calculate the liquid flow field with localized heating condition. It solves both the pressure and the velocity distributions of the liquid over the whole pipe, thus allowing determination of the pressure gradient and examination of the effect of the location and the geometry of the heater on the heat pipe performance. In the analytical model, the working fluid is assumed to be evaporated uniformly over the heat input zone and also to be condensed uniformly over the condensation section. The method is applied here to a flat plate heat pipe, on which the heater is a rectangular patch and the rest of the pipe is all considered to be the condensation section. In practical applications this may be corresponding to an electronic chip being directly attached to the surface of a flat plate heat pipe and the rest of the pipe simply exposed to the ambient to dissipate the heat absorbed from the chip. The adiabatic section, which is normally available in a heat pipe, has been omitted in the present study, but this section can easily be included in the analysis by modifying a source distribution function.

2. FORMULATION AND SOLUTION

It is assumed that the flat plate heat pipe is of a $2a \times b$ rectangular dimension and is in an x–y plane, as illustrated in Fig. 1. The heat input zone on the plate is a $2c \times (h-d)$ rectangle which occupies an area of $-c \leqslant x \leqslant c$ and $d \leqslant y \leqslant h$, and is shown by the shaded area in Fig. 1(a). The working fluid in the pipe is assumed to be evaporated uniformly over the heat input area at an evaporation rate α^- (kg s^{-1} per unit area), while over the rest of the plate the fluid is uniformly condensed into the plate at a rate α^+ (kg s^{-1} per unit area). Since the arrangement of the heat pipe and the heat input area is symmetric about the y-axis in our study, only half of the plate ($x > 0$) will be considered, and

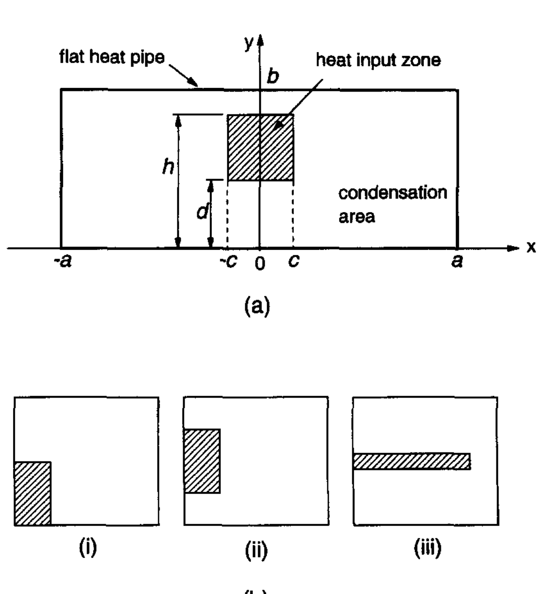

Fig. 1. (a) A schematic illustration of the flat plate heat pipe under local heating. (b) Three typical geometries and locations of the heat zone on the heat pipe.

NOMENCLATURE

a	half length of the heat pipe [m]
A_{mn}	Fourier coefficients of p
b	width of the heat pipe [m]
c	half length of the heating zone [m]
d	y-coordinates of the heating zone [m]
h	y-coordinates of the heating zone [m]
$f(x,y)$	distribution function of the condensation rate
F_{mn}	Fourier coefficients of $f(x,y)$
K	wick permeability
$\dot{m}$	total mass flow rate [kg s^{-1}]
L_e	effective length [m]
p	pressure [N m^{-2}]
p_{ref}	reference pressure [N m^{-2}]
P	non-dimensional pressure, $= (p-p_{ref})/(\beta ab)$
P_{min}	minimal pressure
ΔP	pressure difference, $= P-P_{min}$
ΔP_{max}	maximum pressure difference
R	geometric parameter, $= c/(h-d)$
S	area of the heating zone, $= c(h-d)$
u	velocity component in x-direction [m s^{-1}]
v	velocity component in y-direction [m s^{-1}]
U	$= u\rho/(\alpha^+ b)$
V	$= v\rho/(\alpha^+ b)$
X	$= x/a$
Y	$= y/b$.

Greek symbols

α	overall condensation rate on the heat pipe [kg s^{-1} m^{-2}]
α^+	condensation rate [kg s^{-1} m^{-2}]
α^-	evaporation rate [kg s^{-1} m^{-2}]
β	$= \mu\alpha^+/\rho K$
η	ratio of the condensation area to the heating area
ρ	mass of the fluid per unit area of the flate plate heat pipe [kg m^{-2}]
μ	viscosity of the fluid [kg s^{-1} m^{-1}].

three typical locations and configurations of the heating area shown in Fig. 1(b) will be discussed later on. The total heat input can be calculated by the known area and the heat flux.

In the area of $0 \leqslant x \leqslant a$ and $0 \leqslant y \leqslant b$, the mass conservation of the fluid requires that

$$\alpha^+[ab-c(h-d)] = \alpha^- c(h-d) \tag{1}$$

so that

$$\alpha^- = \alpha^+\left[\frac{ab}{c(h-d)}-1\right] = \alpha^+\eta \tag{2}$$

where

$$\eta = \frac{ab}{c(h-d)}-1 \tag{3}$$

which is the ratio of the condensation area to the evaporation area. The fluid evaporated from the heat input area may be considered to be condensed into the area at a negative condensation rate $-\alpha^-$, as such, the condensation rate over the whole plate can be written as

$$\alpha = \alpha^+ f(x,y) \tag{4a}$$

with

$$f(x,y) = \begin{cases} 1 & c \leqslant x \leqslant a \quad 0 \leqslant y \leqslant b \\ -\eta & 0 \leqslant x \leqslant c \quad d \leqslant y \leqslant h \\ 1 & 0 \leqslant x \leqslant a \quad 0 \leqslant y \leqslant d \quad \text{or} \quad h \leqslant y \leqslant b \end{cases}. \tag{4b}$$

If adiabatic sections are to be inserted into the plate, the source distribution function $f(x,y)$ will be modified to be equal to zero at the adiabatic area. The plate is horizontally positioned and the gravity force on the fluid is assumed negligible. The fluid is in a porous material and it follows Darcy's law. The velocity normal to the vapor–liquid interface is assumed to be zero and the governing equations for the fluid can therefore be expressed by

$$u = -\frac{K}{\mu}\frac{\partial p}{\partial x} \quad v = -\frac{K}{\mu}\frac{\partial p}{\partial y} \tag{5a}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = \frac{\alpha}{\rho} \tag{5b}$$

in which p is the pressure of the fluid, u and v are the flow velocities in x and y directions, respectively, K is the permeability of the wick structures, μ is viscosity, ρ is the mass of the fluid per unit area of the plate, and α is the condensation rate in equation (4a). The combination of equations (5a) and (5b) yields an equation for the pressure p

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2} = -\beta f(x,y) \tag{6}$$

where $\beta = \mu\alpha^+/\rho K$. The boundary conditions for the pressure are

$$\left.\frac{\partial p}{\partial x}\right|_{x=0} = \left.\frac{\partial p}{\partial x}\right|_{x=a} = \left.\frac{\partial p}{\partial y}\right|_{y=0} = \left.\frac{\partial p}{\partial y}\right|_{y=b} = 0 \tag{7}$$

according to which the pressure can generally be expressed in a Fourier's series:

$$p = \beta ab \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

$$= p_{ref}+\beta ab\left\{\sum_{m=1}^{\infty} A_{mo}\cos\frac{m\pi x}{a}+\sum_{n=1}^{\infty} A_{on}\cos\frac{n\pi y}{b}\right.$$

$$\left.+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\right\}. \tag{8}$$

In equation (8) p_{ref} is the reference pressure of the fluid, the coefficients A_{mn} will be determined by substituting equation (8) into equation (6). This gives

$$ab\left\{\sum_{m=1}^{\infty} A_{m0}\left(\frac{m\pi}{a}\right)^2 \cos\frac{m\pi x}{a} + \sum_{n=1}^{\infty} A_{0n}\left(\frac{n\pi}{b}\right)^2 \cos\frac{n\pi y}{b} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\right\} = f(x,y). \quad (9)$$

The source distribution function $f(x,y)$ at the right-hand side of equation (9) can be expanded in the same Fourier's series, i.e.

$$f(x,y) = \sum_{m=1}^{\infty} F_{mo}\cos\frac{m\pi x}{a} + \sum_{n=1}^{\infty} F_{on}\cos\frac{n\pi y}{b} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} F_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \quad (10)$$

where

$$F_{mo} = -\frac{(h-d)a}{m\pi}(1+\eta)\sin\frac{m\pi c}{a} \quad (11a)$$

$$F_{on} = -\frac{cb}{n\pi}(1+\eta)\left[\sin\frac{n\pi h}{b} - \sin\frac{n\pi d}{b}\right] \quad (11b)$$

$$F_{mn} = -\frac{ab}{(m\pi)(n\pi)}(1+\eta)\sin\frac{m\pi c}{a}\left[\sin\frac{n\pi h}{b} - \sin\frac{n\pi d}{b}\right]. \quad (11c)$$

By substituting equation (10) into (9) and comparing the coefficients at both sides, the coefficients A_{mn} are found as

$$A_{mo} = \frac{2}{b^2(m\pi)^2}F_{mo} \quad (12a)$$

$$A_{on} = \frac{2}{a^2(n\pi)^2}F_{on} \quad (12b)$$

and

$$A_{mn} = \frac{4}{b^2(m\pi)^2 + a^2(n\pi)^2}F_{mn}. \quad (12c)$$

By setting

$$P = \frac{p - p_{\text{ref}}}{\beta ab} \quad U = \frac{u}{\alpha^+ b/\rho} \quad V = \frac{v}{\alpha^+ b/\rho} \quad X = x/a$$

and $Y = y/b$

we have

$$P = \sum_{m=1}^{\infty} A_{mo}\cos(m\pi X) + \sum_{n=1}^{\infty} A_{on}\cos(n\pi Y) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\cos(m\pi X)\cos(n\pi Y) \quad (13)$$

$$U = -\frac{\partial P}{\partial X} = \sum_{m=1}^{\infty} A_{mo}(m\pi)\sin(m\pi X) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}(m\pi)\sin(m\pi X)\cos(n\pi Y) \quad (14)$$

$$V = -\frac{a}{b}\frac{\partial P}{\partial Y} = \frac{a}{b}\left\{\sum_{n=1}^{\infty} A_{on}(n\pi)\sin(n\pi Y) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}(n\pi)\cos(m\pi X)\sin(n\pi Y)\right\}. \quad (15)$$

The pressure field and the velocity distribution can be calculated by the above three equations. Results will be presented in the next section.

3. RESULTS AND DISCUSSION

The pressure and the flow velocities are evaluated in this section to show the effect of location and geometry of the heat input area on the distribution of the pressure and the pressure drop across the plate. The infinite series are truncated at $m = n = 40$ in the numerical computation. In following discussion the geometry of the plate is set as $a = b$, and the area of heat input zone is fixed at $c(h-d) = ab/8$. The dimensions and the geometry of the heat input patch are initially fixed at $c = a/4$, $h-d = b/2$ and $c/(h-d) = 1/2$, whereas the location d varies from $d = 0$ to $d = b/2$. The location of the heat input area is then fixed at the centre of the plate while the geometric parameter $c/(h-d)$ varies from 1/8 to 8. Figure 1(b) illustrates three typical dimensions and locations of the heating zone in the calculation. The calculated pressure fields are illustrated by contours of pressure difference $\Delta P = P - P_{\min}$, where $P_{\min}$ is the minimal pressure in each case. Figure 2 shows the computational results for the arrangement illustrated in Fig. 1(b). It can be seen that the lowest pressure is always at the inner part of the heat input area whereas the highest pressure is always at the region the most distant from the heating area. The velocity fields [Figs. 2(a2), (b2) and (c2)] illustrate clearly that the flow velocity increases from the high pressure corner to the heating zone, and decreases after reaching the heater. This shows that the fluid is condensed from the cold area and evaporated into the heating area. Figure 2(b1) also shows that maximum pressure drop across the plate becomes $\Delta P_{\max} = 0.48$, which is smaller compared to the pressure drop $\Delta P_{\max} = 0.68$ when the heater is placed at the side, shown in Fig. 2(a1). For different heater positions but the same heater geometry, as illustrated in Figs. 1(i) and 1(ii), $\Delta P_{\max}$ is computed and plotted against d in Fig. 3. It shows that the maximum pressure difference has a minimal value at $d = b/4 = 0.25$. At this position it will produce a minimal pressure drop for the same flow rate (note that the condensation area is fixed and the condensation rate is a constant). In other words, when the heat source is placed at the middle of the plate, the capillary heat transport limit will be at its maximum. This conclusion is consistent with the results obtained by the approximate method based on the effective length of heat pipe [4].

The heater is now placed at the middle of the plate and stretched along y-direction with its area being fixed at $ab/8$ and the geometric parameter $c/(h-d)$ varied from 1/8 to 8. Figure 2(c1) shows that the maximum pressure drop is reduced further in this case. $\Delta P_{\max}$ is computed for different ratio $c/(h-d)$ and the results are plotted as the solid line in Fig. 4. It is seen that, in general, the bigger value of $c/(h-d)$ will give less pressure drop. This is probably due to the longer circumference of the heating area, across which the flow velocity will be smaller for the same total mass flow, as shown by Fig. 2(c2), and therefore require less driving pressure.

The pressure drop calculated by the analytical method is finally compared with the estimated results based on the effective length. The effective length method has been applied by Schmalhofer and Faghri [3] to a blocked-heated heat pipe, and by Sun *et al.* [4] to a partially heated flat plate heat pipe, in which the heat flow rates were assumed to be uniform across both the condensation sections and the evaporation sections. The same technique is adopted in the present case, in which the rectangle heater is located at the middle of the y-dimension of the heat pipe, such as the situation illustrated by Figs. 1(ii) and 1(iii). The results by Sun *et al.* [4] have been modified here to take into account the situation that the heater is surrounded by the condensation region rather

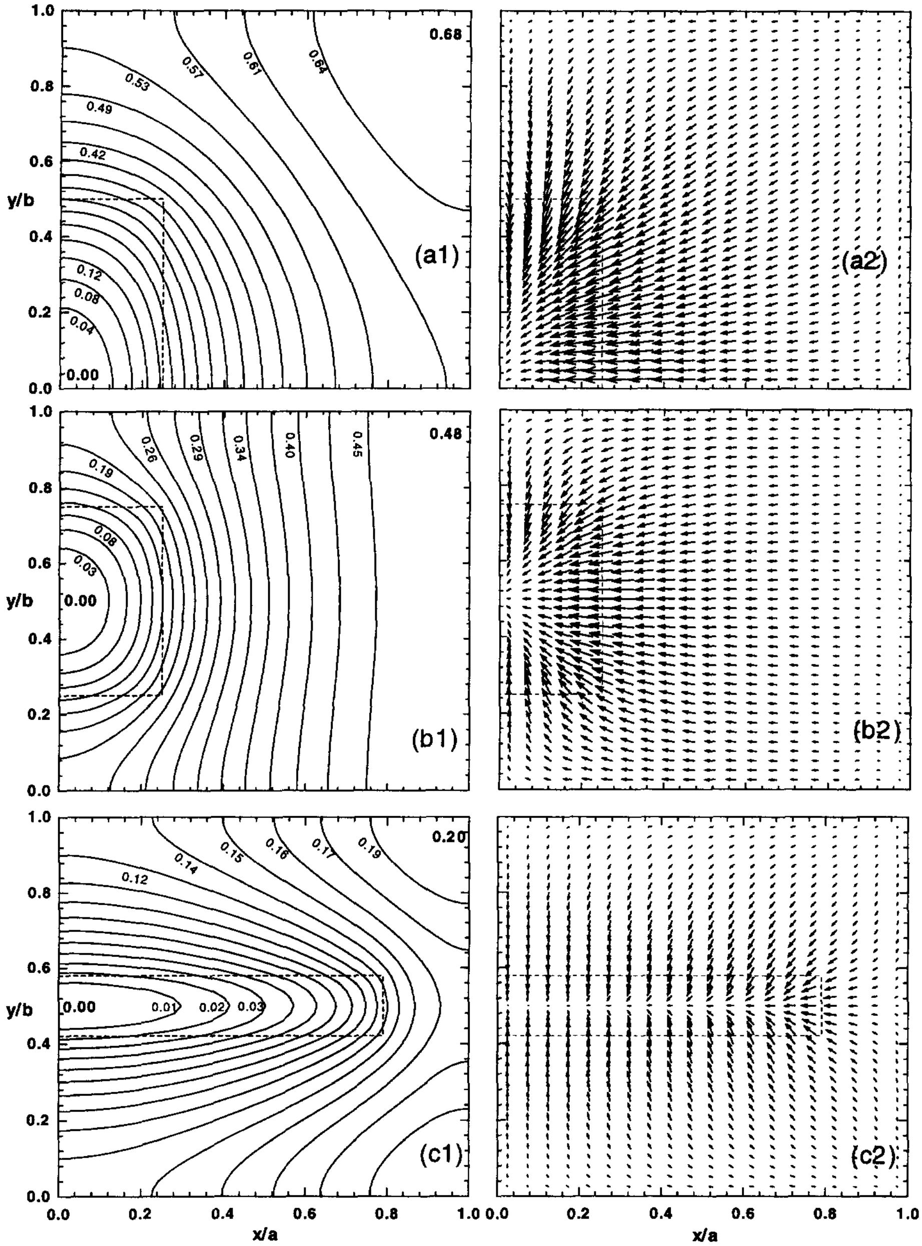

Fig. 2. Pressure contours of $\Delta P = P - P_{\min}$ and the flow velocity fields corresponding to three configurations shown in Fig. 1(b). The dashed lines indicate the heating zone.

than being separated from it by an insulation section. The effective length L_e in the present case is obtained as

$$\frac{L_e}{b} = \frac{1}{2[1-S/(ab)]}\left\{\frac{a}{b} - \sqrt{\left[\frac{SR}{a^2}\right]} + \frac{1}{4}\left(1-\sqrt{\left[\frac{SR}{a^2}\right]}\right)\left(1-\sqrt{\left[\frac{S}{b^2R}\right]}\right) + \frac{1}{4}\left(\sqrt{\left[\frac{SR}{a^2}\right]} - \frac{S}{a^2}\right)\right\}. \quad (16)$$

The effective length is associated with the pressure drop by [1, 3]

$$\Delta p = \frac{\mu \dot{m}}{K\rho} \times \frac{L_e}{b} \quad (17)$$

where $\dot{m}$ is the total mass flow rate in the wick structure. In the present case,

$$\dot{m} = \alpha^- c(h-d) = \alpha^+ \eta c(h-d). \quad (18)$$

By substituting equation (18) into equation (17) and taking $\beta = \mu\alpha^+/K\rho$, $P = \Delta p/ab\beta$, the estimated maximum pressure drop is obtained by

$$\Delta P_{\max} = \frac{\eta S}{ab} \times \frac{L_e}{b}$$

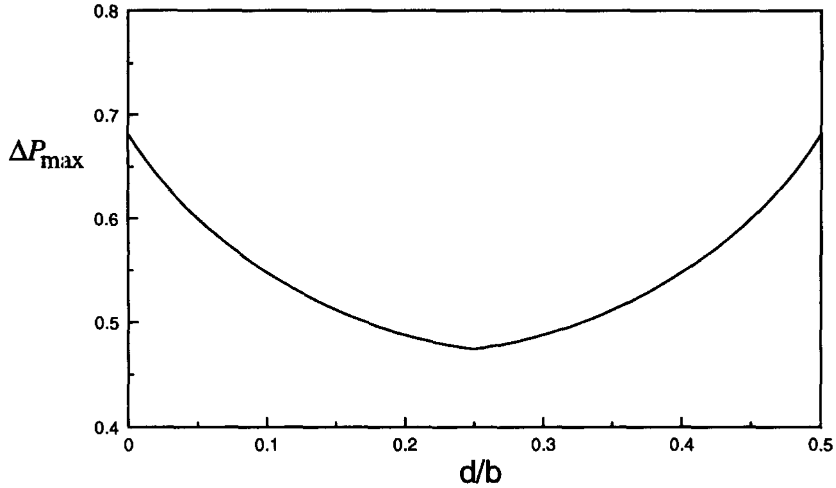

Fig. 3. The maximum pressure drop across the plate vs the position d/b. The dimension of the heater is $c/a = 1/4$, $(h-d)/b = 1/2$ and the geometric parameter is $c/(h-d) = 1/2$.

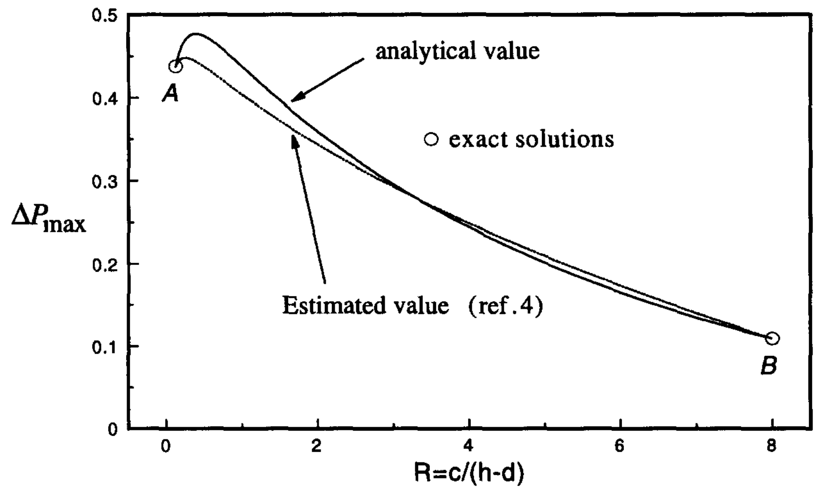

Fig. 4. The maximum pressure drop vs the geometric parameter $R = c/(h-d)$. The solid line is the calculated pressure drop, the dotted line is the estimated pressure drop based on the effective length. The heater is placed at the centre of the plate similar to Fig. 1(iii).

$$= \frac{\eta S}{ab} \times \frac{1}{2[1-S/(ab)]} \left\{ \frac{a}{b} - \sqrt{\left[\frac{SR}{a^2}\right]} + \frac{1}{4}\left(1 - \sqrt{\left[\frac{SR}{a^2}\right]}\right)\left(1 - \sqrt{\left[\frac{S}{b^2 R}\right]}\right) + \frac{1}{4}\left(\sqrt{\left[\frac{SR}{a^2}\right]} - \frac{S}{a^2}\right) \right\}. \tag{19}$$

Equation (19) was evaluated for the same dimensions of the heater and the heat pipe as in the previous calculations, i.e. $a = b$, $S = ab/8$, and the results are presented in Fig. 4 by the dotted line. It is seen that the estimated pressure drops based on the effective length method are generally close to the calculated values, especially at points A and B, the estimated pressure drops are equal to the calculated values. At the point A, $R = c/(h-d) = 1/8$, $c/a = 1/8$ and $(h-d)/b = 1$, i.e. the heater is a strip completely across the heat pipe in the y-dimension. At the point B, $R = c/(h-d) = 8$, $c/a = 1$ and $(h-d)/b = 1/8$, i.e. the heater is a strip completely across the heat pipe in the x-dimension. In both cases the heat flows are actually 1D and uniform as assumed by the method, so that the estimated results are the same as the exact solutions at these two points. The results indicate that the present analytical solution can give reliable results on the maximum pressure drop of the working fluid in the wick structure of a flat plate heat pipe with localized heating condition.

4. CONCLUDING REMARKS

An analytical study has been conducted to examine the effect of a localized heating on the flow of liquid in a flat plate heat pipe. The pressure and the flow velocity are studied in detail with different locations and the geometrical configurations of the heater. The results show that, in terms of the minimal pressure drop across the wick structure, the optimal location is at the geometric centre of the heat pipe surface and the optimal geometry, for a constant heat input, is of the longest circumferential length of the heater. The technique presented here demonstrates a better way to deal with discretely distributed heat sources. This method, in principle, can be used to calculate the pressure filed for multiple heaters which are arbitrarily placed on a heat pipe.

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